# TWO APPLICATIONS OF SIMULATION IN HEALTHCARE DECISION MAKING

An Undergraduate Research Scholars Thesis

by

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# ABSTRACT

Two Applications of Simulation in Healthcare Decision Making



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This study investigated the use of simulation modelling to promote informed decision making in two contexts: healthcare operations and classroom education. The first of these relied on a simple game-based model under a lean framework to aide healthcare stakeholder in the identification of overly variable processes. By creating a validation model in Microsoft Excel, we were able to create a communicable tool to assist in the universal implementation of lean methodologies. The second context identified the Monty Hall Problem as an efficacious example in helping decision makers overcome cognitive biases in decision making. We argue that the synthesis of this problem with simulation modelling can be used as a tool to teach three concepts: conditional probability, simulation, and informed decision making. Both parts of this investigation add to our understanding of the uses of simulation to promote decision making.

## ACKNOWLEDGEMENTS

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# NOMENCLATURE

| | |
|---|---|
| MHP | Monty Hall Problem |
| MCM | Monte-Carlo Methods |

# SECTION I

# INTRODUCTION AND LITERATURE REVIEW

## Simulation Methodology and Lean in Healthcare

The pursuit of identifying and improving healthcare quality has persisted for decades with many different management frameworks proposed to meet this challenge. One that has consistently held the attention of healthcare management professionals is lean. Emerging in the 1990's, lean has become one of the dominant frameworks for healthcare quality improvement. In practice, lean focuses on the reduction of process waste, the continuous improvement and standardization of work processes, and the lowering of inventory overhead. These methods have been incredibly successful in improving specific health institutions (D'Andreamatteo et al. 2015). While lean has proven beneficial in many cases, much work remains in standardizing and universalizing the implementation of lean throughout the healthcare systems (Robinson et al. 2012).

Many issues have faced the implementation of lean in healthcare with researchers noting the difficulty in teaching a lean mindset (Maijala 2018) and the inability to standardize the lean over different health systems (Holden et al. 2015) among other reasons. Most troubling, researchers have pointed to a lack of stakeholder engagement as one of the primary reasons for lean's disjointed and varied implementation and success (Robinson et al. 2012; Hamrock et al. 2013; Robinson et al. 2014).. While stakeholders should be engaged in any management system, a lack of sustained engagement is especially damaging to lean as it requires all process stakeholders to continually work together to develop novel strategies for work improvement. In this way, lean functions as a "bottom up" methodology in which every member is empowered to change issues encountered through their work, making it imperative that the ideas of lean are understood. This requires techniques that can merge methodologic validation and education for successful implementation of lean management philosophies.

To promote these standards, a growing area of interest among management researchers has been the intersection of simulation and lean methodologies in healthcare (Radnor et al. 2010; Robinson et al. 2012). These studies have noted the synergism between a joint simulation and lean approach for alleviating a range of issues in healthcare; though, these methodologies are typically implemented independently of each other with their combined benefits going unnoticed.

To address these challenges, our study builds upon previous work to use simulation as a tool for validate lean principles and stakeholder understanding of lean (Robinson et al. 2012; Hamrock et al. 2013; Robinson et al. 2014). By creating and analyzing a simulation model based upon a well-known children's game and tying the game to health services delivery, we illustrate how this model can simultaneously validate lean principles and enhance understanding of the effect of lean principles on process improvement. Moreover, our model contributes to the need for educational materials to illustrate lean methodologies.

**Review of Relevant Literature**

For two decades, lean principles have been a prime focus for management professionals and researchers. First defined in the late 1990's, Lean and Six Sigma management practices have been lauded for their ability to revitalize and reconstitute failing production methods and institutions. Examples of their successful implementation exists in a wide spectrum of organizations, from their inception in car manufacturing at the Toyota company (Womack and Jones 2003) to recent examples in financial services (Li, Field, and Davis 2018). Womack and Jones (2003) were among the first to codify the lean mindset in an organized set of principles.

These principles orient management around a holistic view of all the determinants of systemic improvement based on constant procedural waste reduction and refinement.

Healthcare managers were quick to recognize the utility of lean in their industry-specific context. Notably, many healthcare institutions have successfully implemented procedures under a lean paradigm to overcome deficits and recent notable examples include Virginia Mason in Seattle and Royal Bolton NHS (Robinson et al. 2012). Additionally, the adoption of lean has been recognized as a way to combat a range of issues like rising hospital costs (Gitlow and Gitlow 2013) and shortages in the blood supply chain (Katsaliaki, Korina, Mustafee, and Kumar 2014). But, a universal adoption of lean has not been widespread causing "pockets of best practice" to develop, leaving the benefits of lean management unsubstantiated (Radnor, Holweg, and Waring 2011).

Much scholarly work has been done to understand the divide between lean's theoretical benefits and its implementation. Some researchers have pointed to the difference between public attitude and professionals understanding of lean. The very term "lean" gives the impression of an "quick fix" methodology as opposed to a long-term mindset (Schonberger 2018). This view can be exacerbated by how lean is communicated, coming with an entirely new 200 word vocabulary, daunting late adopters with a steep learning curve (Schonberger 2018). Additionally, Radnor, Holweg, and Waring (2011) argue that the issues surrounding lean's adoption in healthcare rest in two factors: first, lean is seen as a management tool focused solely on waste reduction, and second, healthcare organizations are unable to commit to managerial changes due to factors such as senior leadership commitment, resource, availability, and financial costs. Further, Gitlow and Gitlow (2013) assert that traditional management methods aid the creation of an environment detrimental to constant improvement due to a tendency to focus on outcomes rather than on the processes that create outcomes. These factors create a concept of lean that is disconnected from its intent; namely, constant, continual improvement championed by management professionals.

Many tools are available to combat the divide in the different conceptions lean management. Of these, modelling seems to be a particularly powerful candidate for fostering professional understanding due to its ability to simplify complex situations in unified, methodological way. In particular, discrete event simulations (DES) are well suited for modelling healthcare processes since many are dependent sequences of events and because of these model's previous effectiveness in addressing healthcare problems (Hamrock et al. 2013). Moreover, simplified models have been useful to identify and understand stakeholder preferences (Holden et al. 2018). Robinson et al. (2012) devised a framework by which DES modelling in healthcare, and in particular lean in healthcare, should occur. Models as assessment tools were proposed to create understanding of different lean aspects. Furthering their work, Robinson et. al (2014) calls for extremely simplified, user-facilitated models to encourage stakeholders, who lack managerial expertise, engagement with the overarching lean mindsets. In essence, to play with the models to learn lean.

In implementation, Six Sigma and lean mindsets are often most concerned with variation in processes. Physical waste is not a main concern of non-capital-intensive processes like those found in healthcare; instead, process runtime can be one of the main factors to be minimized. Gitlow and Gitlow (2013) identify special variation in processes as a key component in rising healthcare costs. Variations in processes are an inevitability; this is especially true for the processes in healthcare setting. Under a traditional paradigm, variation can be acceptable if the outcome leads to the process closer to its target outcome. Lean rejects this concept because process efficacy requires consistently meeting target outcomes, so lean methodologies are uniquely suited to combating rising costs. This study describes a DES model that can: 1) theoretically identify areas for improvement in processes and 2) serve as a training tool to show how to differentiate deficits in procedural variation from that inherent variation.

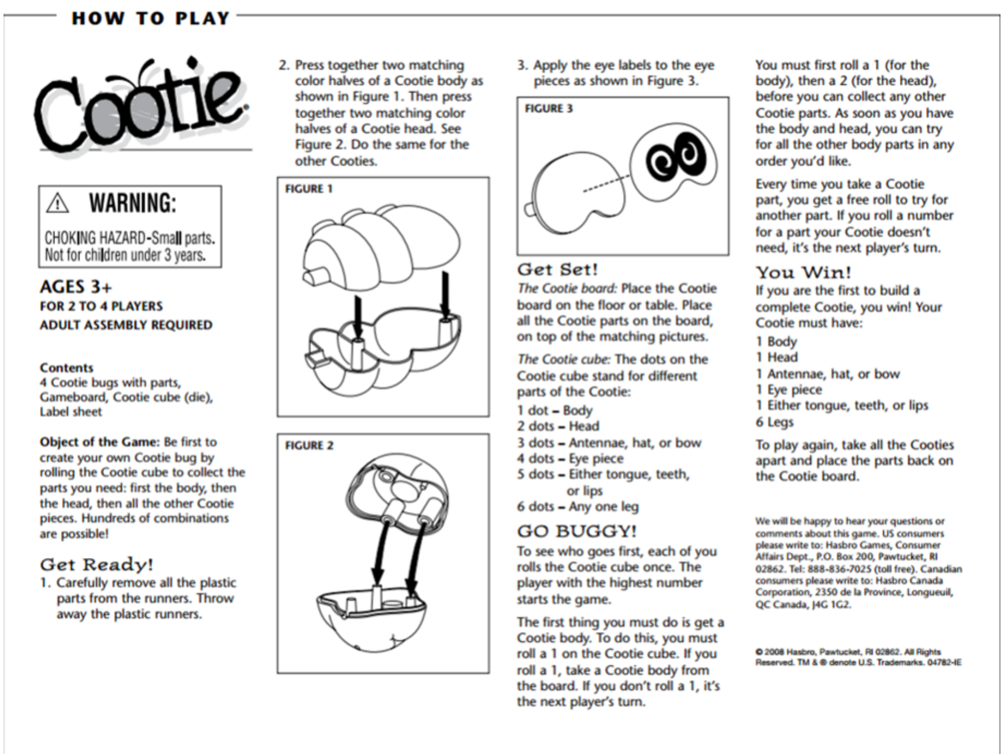
**Game Simulation Methods and Analysis**

Procedural thinking is an important aspect of healthcare improvement. The term process is defined by Evans and Lindsay (2017) as "a sequence of linked events that is intended to achieve one result." The majority of activities undertaken by organizations are processes, which can be broadly classified into either internal or external value creative or supporting processes. Since these represent most of the work done by organizations, both an understanding of processes and careful process management are essential to successful operations. Process management is often categorized into three distinct sequences: design, control, and improvement. In the design phase, the goals and value inputs of are determined, and a process is created to reflect all the values and needs of all process stakeholders. Traditionally, these requirements are systematically codified and used to create a visually representative process map that allows for each step and the relationship between each step to be accounted for by the process designers. This mapping gives valuable insight to both the function and flow of the desired process. Normally, the next aspect of process management is control. In this stage, managers ensure that processes constantly achieve their minimal requirements. These requirements are defined in terms of process outcomes, typically the quality of the output, and the variation in product outcomes.

Lean management focuses on the third and final area of procedural management, procedural improvement. Under the lean mindset, processes are considered holistically in terms of all determinants of process effectiveness. Lean often relies on the implementation of kaizen, or small gradual improvement, into management practices, but the implementation of these practices requires accessible educational tools. Building on previous work to integrate simulation modelling and lean management (Robinson et al. 2012; Hamrock et al. 2013; Robinson et al. 2014 ), we create and analyze a model that conceptualizes process flow through a well-known children's game, Cootie. Because simulation models are an efficacious way to understand the variants of process flow, their use is fruitful for everyone engaged in a process since a culture of continuous improvement requires consistent, educated input from all.

Children's games provide an effective simplistic modeling opportunity to promote educational endeavors. Two main attributes go into making children's games likely candidates for modelling. First, these games are normally short given their intended audience. This quick run time allows for modelers to replicate the game easily and efficiently, allowing for stochastic models to be created in an intuitive manner. Second, the rules of these games are, by their nature, easy to understand again due to their intended audience. This property allows for the easy codification of modelling assumptions based directly on the rules of game allowing for model users to see the link between scenario parameters and model performance. Even inexperienced users will be able to quickly comprehend and engage with the theoretical setting of the model. In this study, these factors make the use of a simple children's game an effective tool for fostering stakeholder engagement, meeting calls by previous researchers for the creation of simple models to highlight facets of lean management practices.

The game Cootie is an effective analogy that can aid healthcare providers with the process management. The purpose of the game to build a Cootie, a bug-like creature depicted in Figure 1, from its component parts; namely, a body, a head, six legs, antennae, eyes, and a mouth. This creation of is done through turn based game process where each player rolls a die with the die value tied to a specific component part, for instance rolling a 6 equates to a Cootie's leg. From there, players can create the Cootie by linking the pieces together. Players are limited from linking pieces together until they have two initial components; the body is required before placing legs together, and similarly, the head is required before placing the antennae and eyes. These rules make Cootie an ideal candidate for creating an event simulation tied to healthcare contexts. There is a clear analogy to the stepped nature of healthcare since tasks in healthcare can be seen as events with underlying time constraints and an implicit sequence of activities. In the majority of healthcare processes, the beginning of a step or task requires the completion of the previous task; a precedence relationship. From the rules of Cootie, the successful completion of a step requires the completion of a prescribed step. The delayed nature of start (i.e., having the head and body pieces before subsequent pieces can be collected) also mimics the random start time found in many healthcare processes like that of a patient arriving to an emergency room. Most importantly for DES modelling, there is an underlying procedural flow between distinct time steps (i.e., turns) which allows for clear measurements of performance in terms of these steps.


HOW TO PLAY

Cootie®

WARNING:
CHOKING HAZARD-Small parts.
Not for children under 3 years.

AGES 3+
FOR 2 TO 4 PLAYERS
ADULT ASSEMBLY REQUIRED

Contents
4 Cootie bugs with parts, Gameboard, Cootie cube (die), Label sheet

**Object of the Game:** Be first to create your own Cootie bug by rolling the Cootie cube to collect the parts you need: first the body, then the head, then all the other Cootie pieces. Hundreds of combinations are possible!

Get Ready!
1. Carefully remove all the plastic parts from the runners. Throw away the plastic runners.
2. Press together two matching color halves of a Cootie body as shown in Figure 1. Then press together two matching color halves of a Cootie head. See Figure 2. Do the same for the other Cooties.

FIGURE 1

FIGURE 2

3. Apply the eye labels to the eye pieces as shown in Figure 3.

FIGURE 3

Get Set!
*The Cootie board:* Place the Cootie board on the floor or table. Place all the Cootie parts on the board, on top of the matching pictures.

*The Cootie cube:* The dots on the Cootie cube stand for different parts of the Cootie:
1 dot – Body
2 dots – Head
3 dots – Antennae, hat, or bow
4 dots – Eye piece
5 dots – Either tongue, teeth, or lips
6 dots – Any one leg

GO BUGGY!
To see who goes first, each of you rolls the Cootie cube once. The player with the highest number starts the game.

The first thing you must do is get a Cootie body. To do this, you must roll a 1 on the Cootie cube. If you roll a 1, take a Cootie body from the board. If you don't roll a 1, it's the next player's turn.

You must first roll a 1 (for the body), then a 2 (for the head), before you can collect any other Cootie parts. As soon as you have the body and head, you can try for all the other body parts in any order you'd like.

Every time you take a Cootie part, you get a free roll to try for another part. If you roll a number for a part your Cootie doesn't need, it's the next player's turn.

You Win!
If you are the first to build a complete Cootie, you win! Your Cootie must have:
1 Body
1 Head
1 Antennae, hat, or bow
1 Eye piece
1 Either tongue, teeth, or lips
6 Legs

To play again, take all the Cooties apart and place the parts back on the Cootie board.

We will be happy to hear your questions or comments about this game. US consumers please write to: Hasbro Games, Consumer Affairs Dept., P.O. Box 200, Pawtucket, RI 02862. Tel: 888-836-7025 (toll free). Canadian consumers please write to: Hasbro Canada Corporation, 2350 de la Province, Longueuil, QC Canada, J4G 1G2.

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**Figure 1: Instructions for how to play Cootie**

To create and analyze these performance measures, we chose to use Monte Carlo simulation because of its ability simulate outcomes of many plays of a given game. In particular, Monte Carlo methods are used to model outcomes based on random number generation by simulating the randomness of the die roll in the Cootie game. Our application allows us to retrieve numeric values that can then be translated to an associated piece of Cootie which, in turn, illustrates how the game might unfold. The specific instructions for the game are as provided in Figure 1.

The summarized results of the simulation determine the performance measures of the game. These performance measures are: 1) the runtime before the games completion (i.e., the number of turns until the Cootie is fully constructed, otherwise known as the game's cycle time), 2) the time to get the head and body (i.e., how long until the player moves beyond the initial process requirements), and 3) how much time is spent in each state of the game state (i.e., how many turns before getting the body piece). Each of these measures has many useful healthcare analogies. For example, 1) Total run time can be seen as the amount of a time a patient is interacting with the system, from the start of the intake process to discharge disposition, 2) The time to get the head and body is comparable to the amount of time it takes for a patient to engage with healthcare processes, for example, the amount of time required to fill in intake forms before being seen by a health professional, and 3) the game state parallels the time after between intake completion and the patient's exit from the health system.

The DES model was created in Excel. We chose to use Excel for two reasons. First, Excel is a software that is ubiquitous among management professionals. Having the DES model on accepted software allows for easy engagement with stakeholders. The second modelling reason is that the random number generation properties of Excel make it efficient for use in Monte Carlo methods. The purpose of the simulation is to discover the behavior of the game under different variant conditions. The variation in the simulation is initially given by the roll of the die. In the standard game, a fair six-sided die is used so that each body part has an equal one out of six chance of being chosen. Our simulation begins with this as a set probability distribution and assignment to establish the model's baseline. We then explore game outcomes by varying the baseline conditions to identify and illustrate how the changing conditions influence model behavior. In turn, these results are summarized and discussed relative to model's baseline.

Figure 2 shows instances of the game under different conditions in four separate cases. We define the baseline as "Case A" as this represents playing the game under the original, intended instructions where the gathering the head and body have to happen before any other piece can be collected. Case B is provides the player with the head and body pieces already given (this is similar to starting the process with a head start). Case C adjusts the probability of the legs to be 50% compared to the $1/6^{th}$ probability as given in the base line. Case D combines Case B and Case D (i.e., the player has a head start and the underlying probability distribution favors the leg pieces). We chose to make the adjustment in Case C (and Case D) because the legs are the most time consuming step in the base line. The Panels A through D in Figure 2 show the percentage of rolls in each game state (i.e., the number of pieces collected toward building the Cootie) along with the cumulative percentage roles for each step in the game process. Figure 3 gives the state conditions for an individual game simulation. Table 1 shows summary statistics of the performance measures for the four cases.

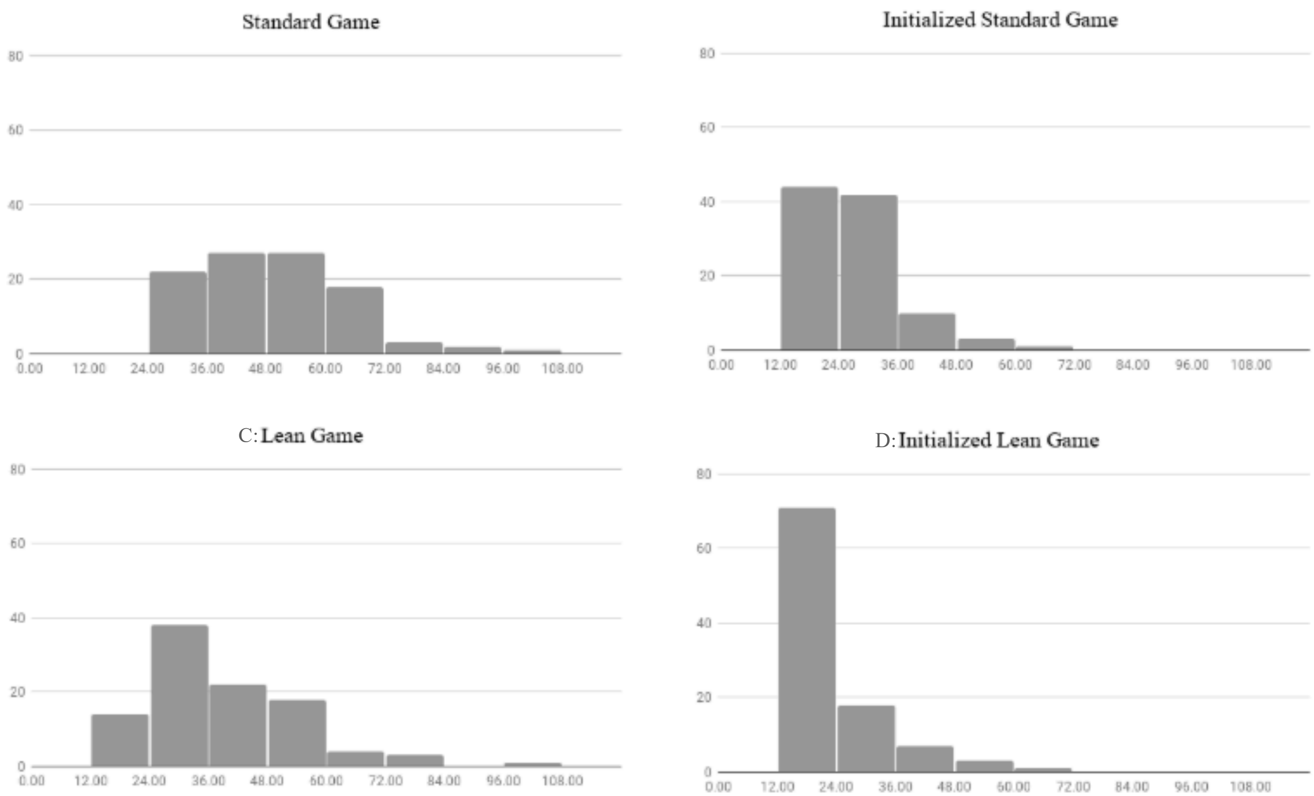


**Figure 2: Histograms of game run times: number of turns until completion.**

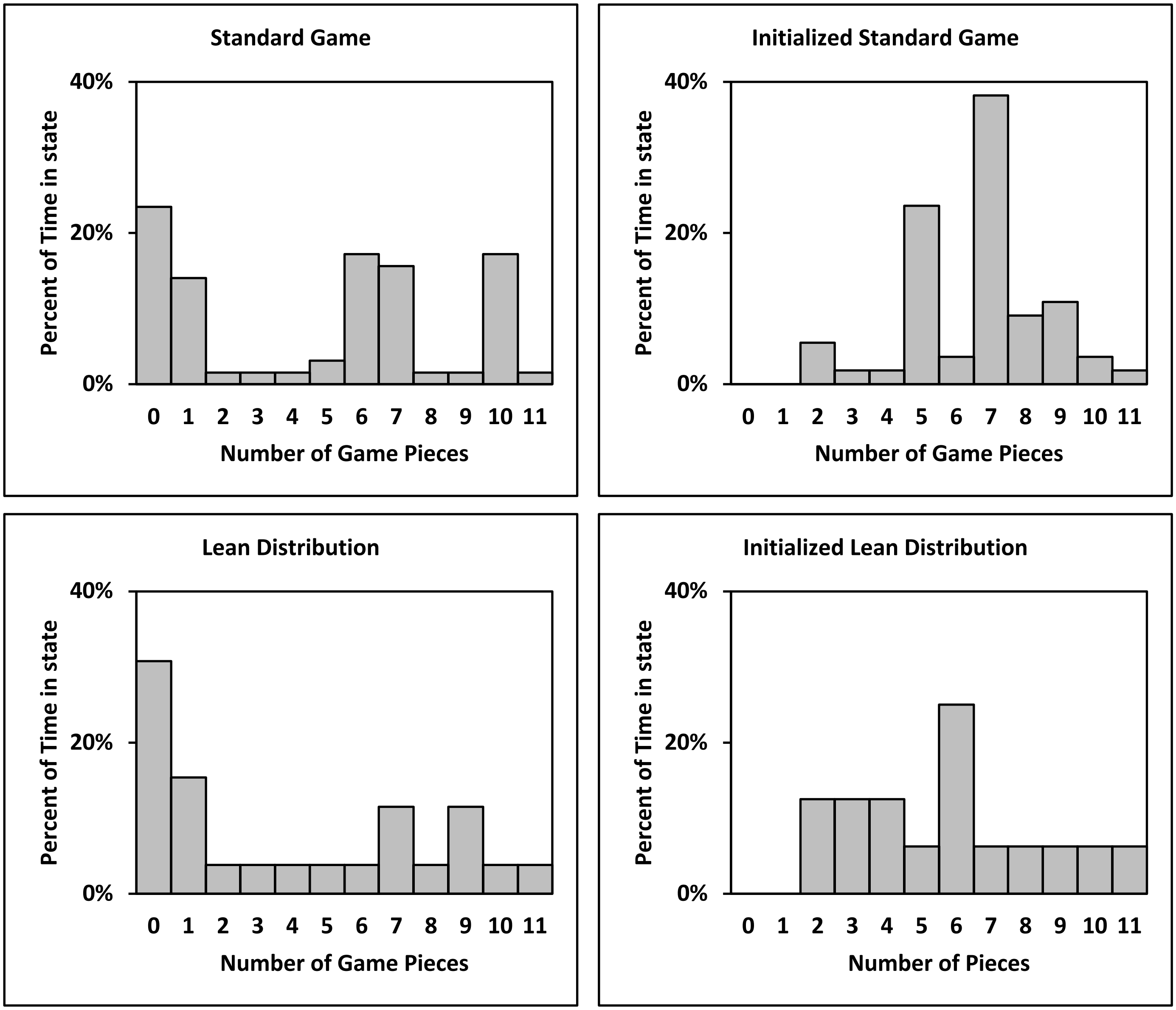


**Figure 3: State distributions in individual game simulations. Each piece represents a state before completion of the game (12 pieces).**

**Table 1: Summary statistics of game run times over 100 repeated trials**.

| Performance Measures | Case A<br>Standard Game | Case B<br>Initialized Standard Game | Case C<br>Lean Game | Case D<br>Initialized Lean Game |
|---|---|---|---|---|
| Mean | 48.31 | 26.28 | 38.58 | 22.51 |
| Standard Deviation | 15.76 | 9.06 | 15.57 | 10.31 |
| Median | 48 | 24.5 | 35 | 19 |
| Minimum | 24 | 14 | 18 | 12 |
| Maximum | 104 | 60 | 99 | 69 |

## Discussion of Results

The results presented in Figure underscore the effectiveness of our methodology on procedural improvement. The change in the underlying variability of the game process, e.g. increasing the likelihood of the most time-consuming portion, drastically shifted the time distribution. There are two useful comparisons to understand this distributional shift: the mean and median runtimes given the first and second row of Table 1 respectively. For each implementation of lean, the mean runtime is largely shorter than its counterpart: a difference of 9.73 turns in the uninitialized cases (A and C) and a difference of 3.77 in the initialized cases (B and D). Even more convincingly, a dramatic shift occurred in the medians of the two comparison groups. Both the uninitialized and initialized group medians were almost halved with the lean game conditions with changes of 23.5 and 16 in runtime. Since the distributions are fairly skewed (apparent from the graphs in Figure 1 and from the game's conditions), the median is the most likely measure of center, so the large reduction in median highlights a large central shift in expected run time. These shifts translate into larger scaled differences of in expectation between games under the two comparison groups.

Since the game is an efficacious analogy for health services procedures, the distributional shift reflects the effect lean management can have on procedural run-times when implemented correctly. Lean's correct implementation has well studied and benefits for all parties who make use of health services processes. Our results highlight one area that managers can easily apply leans concepts. By identifying components of largest variation, managers will be able to target process components that most effect overall process quality.

## Conclusion

The key aim for lean management is to develop systems that are balanced and consistently improving, and to empower stakeholders to become actors in procedural improvement. The merits of lean have been proven many times, but work remains to adopt lean throughout the majority of healthcare systems. Undertaking the challenge of implementation, lean in healthcare contexts requires tools that promote a wider understanding of lean in the face of its detractors and the natural reticence of its beneficiaries, process stakeholders. These tools must be readily accessible and interpretable to engage with these stakeholders who are experts in their domains but might not understand the lessons afforded by procedural thinking.

In this paper, we present one such tool that accomplished in our two goals: (1) to validate the lean methodology in terms of process improvement and (2) to contribute to an educational model to aide educate for lean's hopeful, overall universal adoption. This tool provides a novel and helpful expansion of simulation methods in healthcare. Our results show an example of lean's ability to work as a guiding force in process improvement. Moreover, our model provides a road map that can be used by managers to benefit their systems. With careful consideration and training, health professionals can become empowered agents in adoption of lean improving healthcare's most vital systems for the benefit of all.

# SECTION II

# THE MONTY HALL PROBLEM AND ITS APPLICATION

## The Monty Hall Problem

The Monty Hall Problem (MHP) is an important and, for many, an often frustrating and counterintuitive probability puzzle. While many variants exist, the standard approach to the MHP is as follows. The game show “Let’s Make a Deal” historically hosted by Canadian Monty Hall presents a player with three doors to choose from where one door there is a sports car (or some highly valuable prize) that the contestant will win if they choose the door correctly. Behind the two other doors are valueless consolation prizes (in the original television series of the game show, these were commonly depicted as livestock). At the start of the game, the contestant chooses one of three doors. However, before revealing the contents of the chosen door, the game show host opens one of the unchosen doors and gives the contestant the option to switch their initial choice to another door. Advice from the members of the studio audience typically follows with recommendations to “Switch!” or “Stay!” This begs the obvious question that should the contestant switch or should they remain with their initial choice? And, subsequently, does switching matter? That is, does switching increase the likelihood of winning the car?

On first glance, the intuitive answer for many is that it does not matter; switching or not switching should have the same probability of revealing the car since it is equally likely behind each door. Some might even say that the purpose of the game is to make participants switch in that way, forcing them to live with the consequences of their action versus inaction. But this type of reasoning illustrates the subtle genius of the MHP. Intuitively, it does not appear to matter whether a door is opened or not. Probabilistically, the contestant is better off switching as they are now more likely to win the car if they switch their initial guess after the host’s reveal. The reasoning behind this lies in new knowledge the participant is given; the host changes the probability of success from the initial 1/3 likelihood to now have a 1/2 chance (an even coin toss), so that now switching wins with a probability, 2/3. While this result is a straightforward application of conditional probability, for many people this does not feel right, the probability of winning changing from 1/3 to 2/3 does not support their intuition.

Even though many students have amazing abilities for comprehension, synthesizing experience and theory, they have trouble grasping foundational concepts that run counter to normal intuition. These veridical problems, where counterintuitive answers are correct, pose a special sort of problem for educators. How do they give students the theoretical underpinnings while also reinforcing for students the connection between that theory and practice? In this instance, how does an educator bridge the gap between the abstract Monty Hall Problem and everyday decisions? This paper presents a structured approach for the classroom and turning the MHP into an experiential learning opportunity for students.

**Review of Past Applications**

Referring to the host of the game show and its famous host, Monty Hall, the MHP was first posed in the journal, *The American Statistician*, in 1970 by Selvin. Famously, this problem gained notoriety in the 1990s when it became the subject of a weekly columnist, Marilyn vos Savant, in which the now standard proposition that switching is the optimal decision was popularized (vos Savant 1990). This argument caused an uproar with many readers ranging from mathematical laymen to PhDs contesting the logic of vos Savant's solution. The academic study of the MHP illustrates the highly contestable and counterintuitive nature of the MHP, which often translates into classroom settings, frustrating academics who see it as a means to teach a variety of probabilistic intuitions and their students who see it as counter to the confirmation bias that supports their decision to switch or not.

Educators in many fields have understood the pedagogical potential of the MHP. The MHP has been explored in depth in regard to reinforcing students understanding of the relationship between conditional probability and decision theory. In management contexts, Umble and Umble (2004) applied it to aiding students to intuit about conditional probability using a simple card-based simulation game. These practitioners used the MHP as a way to introduce decision trees to the classroom. But after realizing arguments relying on decision trees seemed unconvincing to students, they developed a game setting to act as reinforcement finding that being given the ability to gain tangible hands-on experience allowed for a beneficial learning experience that connected the MHP to real decisions. That study exemplified the usefulness of the MHP.

Furthering this, psychology researchers have tried to uncover the psychological underpinning of the discomfort surrounding the MHP. For many, the frustration with the game lies with the "psychological illusion" (Bennet 2018) of the game where individuals are more likely to remain on their initial choice for fear that they will regret the shift. This bias is strengthened when participants are "confirmed" in their choice by using the game's outcome to confirm their preexisting "knowledge"' that switching has no value or influence on the game's outcome. Biases of this form are termed as confirmation bias. These two cognitive biases are harmful when they cause actors to ignore information and make poor choices. Consequently, a deep appreciation of the MHP and the ability to generalize its lessons are critical for students and their development as decision makers.

While the MHP helps gives students a theoretical appreciation, different techniques are needed to cement understanding. Among these, computer-based simulation techniques seem the most likely candidate for success. Past research has pointed to the importance of simulations for students in preparation of a career (Giovanni 2018). Developing students' abilities to perform decision analyses on computers is especially critical do to the growing reliance of management positions on quantitative tools and the increasingly complicated choices that need to be made (Clemens 2001). To help students overcome these obstacles to learning, strategies and techniques must be developed to help students understand answers to counterintuitive questions.

**Background Information**

Students often struggle with answering the MHP correctly. Previous data collected by the authors show the independence of demographic factors to answering the MHP. Regardless of gender or age, students more often than not say switching does not matter (or sensing a trick by the host might answer correctly without a fundamental understanding of the underlying principle). The demographic of the students along with response is shown in Table 2 and shows much the same as the historical problem around the MHP; when put to the question of switching or staying, the majority of students answered to stay. Table 2 shows the number of student answers to the question of switching, surveyed from an Master of Healthcare Administration over the last 6 years.

**Table 2: Student Answers to Monty Hall Problem**

| Gender\Switch | No | Yes | |
|---|---|---|---|
| Female | 60 | 10 | 70 |
| Male | 40 | 10 | 50 |
| | 100 | 20 | 120 |

Many students will argue that the choice should not matter usually because their initial choice had an equal, 1/3, chance of picking the correct door. Then, switching or not should not make a difference because the 1/3 chance has turned into a 1/2 chance. This type of reasoning is demonstrably false but is typically explained in ways that can seem unconvincing to those wrestling with the implications of the MHP. A typical way that is used to demonstrate the efficiency of switching follows from a decision tree; one such the figure is shown below as

Figure 4, adapted from Lucas, Rosenhouse, and Schepler (2009). With this graphical representation, the benefit of switching the can be easily seen: instead of the student's intuition of an equal 50-50 chance, there is actually a 2-in-3 chance of the non-initial door if the contest elects to switch doors after the host opens a door.

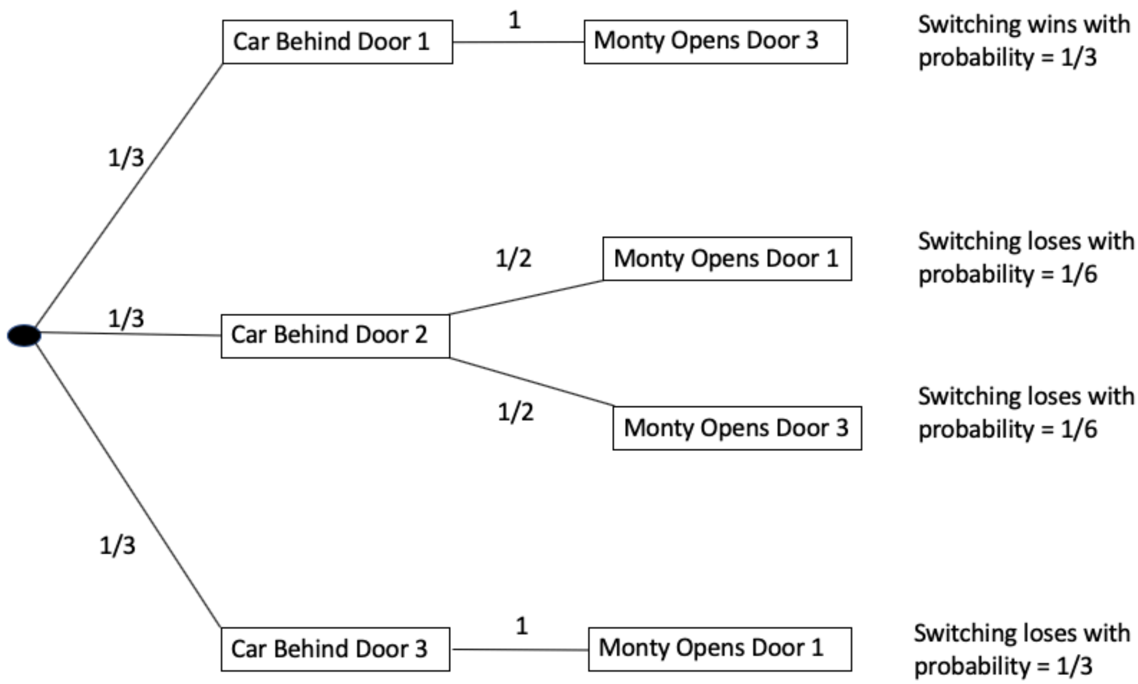


**Figure 4: Probability Tree for MHP when player initially chooses door 2**

The difficulty in students recognizing and being convinced by this sort of argument is the same one that troubled vos Savant's readers. The new information gives contestants the opportunity for a more informed decision. This reasoning follows directly from arguments that center on derivations of Bayes' Rule. The original posing of the question gave another useful graphical argument for the MHP that enumerates all possible outcomes (recreated from Selvin et al. as Figure 5). But, no matter how valid these arguments are, they often do not feel right to contestants in the MHP.

| Car is behind door | Player chooses door | Monte Hall opens door | Contestant Switches | Result |
|---|---|---|---|---|
| 1 | 1 | 2 or 3 | 1 for 2 or 3 | loses |
| 1 | 2 | 3 | 2 for 1 | wins |
| 1 | 3 | 2 | 3 for 2 | wins |
| 2 | 1 | 3 | 1 for 2 | wins |
| 2 | 2 | 1 or 3 | 2 for 1 or 3 | loses |
| 2 | 3 | 1 | 3 for 2 | wins |
| 3 | 1 | 2 | 1 for 3 | wins |
| 3 | 2 | 1 | 2 for 3 | wins |
| 3 | 3 | 1 or 2 | 3 for 1 or 2 | loses |

**Figure 5: Tabular representation of MHP decisions (Selwin et al. 1975)**

Often this counterintuition comes from two challenges facing informed decision making: (1) confirmation bias and (2) a reliance on a sunk cost. The first occurs when actors conceive of new information as part of preconceived beliefs, selectively interpreting it under some invalid framework. In regards to the MHP, this strikes contestants, and students, to feel that Monty opening the door changes the original chance distribution of the door and effects their decision; instead of considering a new 50-50 chance, the choice is conceived in terms of the original distribution. The second of these challenges arises from actor's tendency to remain with their original choice. Actors thinking consequentially of their choices over non-choices is a well understood psychological phenomenon; more intense feelings of failure arise when a choice of moving away from the status quo is made (Massad, Costa dos Santos, da Rocha, and Stupple 2018). In the context of the MHP, contestants tend to want to stand their ground and remain on their initial choice since switching incorrectly feels worse than initially choosing incorrectly. Often, this is reflected by statements like “My first choice is the best choice” and “I am going to stick with the first choice.” Clearly, both these biases are dangerous to well informed decision making.

Luckily, powerful tools are available to analyze decision making strategies, which can be employed to transforming the MHP into an educational endeavor. Microsoft Excel is an invaluable software that can demonstrate the correct decision of switching without relying on analytic arguments. First, simulation in Excel can be used to demonstrate an solution that is non-analytic. Instead of relying on the above decision tree or tabular arguments as shown in Figure 1 and 2, students are able to develop their own coded solution into using Monte Carlo simulation.

We note that this simulation can be a highly desirable skill in the business world since many decisions will not necessarily lend themselves to straightforward analytic analysis. This is doubly important in that a simulated solution will allow for students to explain the differences between the transient and steady state of a problem, an important component of simulation analysis. Much of the intuitive gap around the MHP rests on an attempt to generalize the transient state to the decision making process. The creation of this solution also gives an appreciation for the random number generation capability within Excel as students will see that using the same random number stream as learners will quickly recognize that they receive the same results when running the solution many times without a change to the underlying seed. These considerations lead to the following questions based on Clemens (2001).

**Assignment**

Suppose you are a contestant on the television game show, “Let’s Make a Deal.” You must choose one of three closed doors, labeled 1, 2, and 3, and you will receive whatever is behind the chosen door. Behind two of the doors is a cow, and behind the third is a new car. Like most people, you have no use for the cow, but the car would be very welcome. Suppose you choose Door 1. Now the host opens Door 2, revealing a cow, and offers to let you switch from Door 1 to Door 3

A. Provide your demographics. You are a female or male (please circle) who is _____ years old.
B. Do you switch? Why or why not?
C. What can you assume about the host's behavior?
D. Using Monte Carlo simulation, simulate 100 plays of the game where we don't switch doors. Under this simulation, how many contestants won the car? What is the likelihood of winning the car from these 100 plays?
E. Using Monte Carlo simulation and the same set of random numbers (to determine the location of the car and the behavior of the host regarding which door is opened) you generated in (D), simulate 100 plays of the game where we switch doors. Under this simulation, how many contestants won the car? What is the likelihood of winning the car from these 100 plays?
F. Why was it important to use the same set of random numbers for the simulation models constructed in (D) and (E)?
G. Based on your estimated probabilities in (D) and (E), which strategy do you prefer? Why? How do your results in (D) and (E) compare with what we determined in class? Explain briefly.
H. Plot the cumulative probabilities for both strategies. After how many contestants do your probability estimates for winning stabilize? That is, when do your simulation models enter their steady state?

**Simulation**

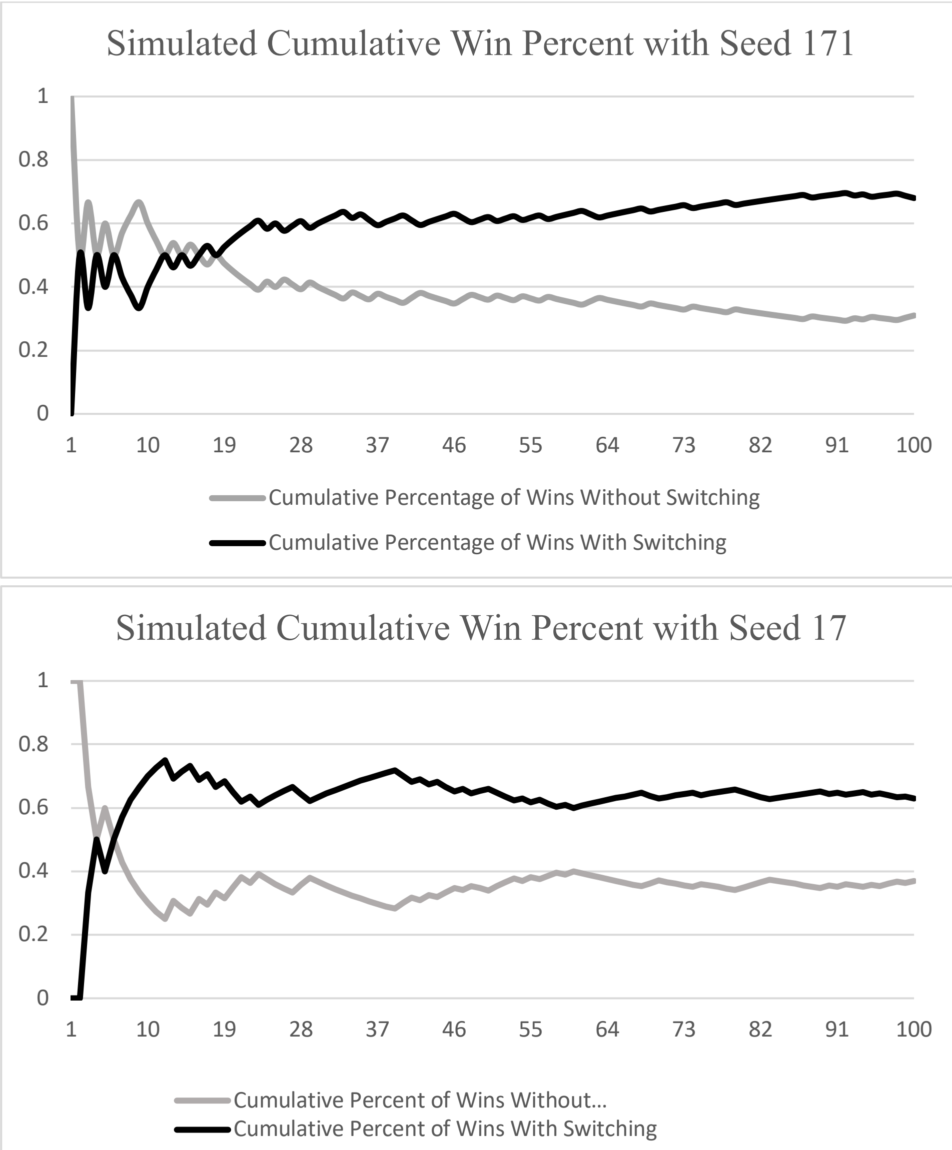


**Figure 6: Simulation Results with two separate seeds (171, 17)**

Figure 6 shows the results of two separate simulation runs providing the desired simulated solution to the Monty Hall problem. Two arbitrary (though it is typically better to use prime numbers due to nature of Excel's seed) seeds were used in Excel to show two divergent solution states. Assigning specific seeds to provides a way to replicate solutions to ensure academic honesty; this also allows for the comparison between simulations noticing that all will begin and end in certain predicable ways. Both simulations begin in transient state, with the cumulative percent seeming to oscillate wildly. This transient state is analogous to the experience of individual players in the game; when making a decision, players only receive one outcome, winning or losing, so they attempt to generalize that to the larger statement about the decision process. For example, a player who switches and loses might say that is evidence against switching. Suppositions like this are shown to be false in the second portion of the simulation graph, the steady state, which verifies the analytic arguments about switching.

Though more generally, these lessons can be interpreted as recommendations about expected action. While individual outcomes may vary significantly from expected results, the long run states match almost exactly the expected underlying probability. So, students, acting as decision makers, are able to contrast what their outcome would have been to the long-term proportion (notice for each simulation the first games each resulted in a win not switching and a loss switching). Statistically, the transition between states and the divide between individual expectation and can be thought in much the same as sampling. With more trials providing ever more information, performance tends towards the expected value of the analytic model. Then, decisions about the performance of long run processes are detached from individual outcomes and intuition.

**A Generalized Extension to the Monty Hall Problem**

A useful rejoinder to extend the lessons of the MHP is to ask: How does increasing the number of doors affect the decision made under new knowledge. Suppose, as vos Savant (1992) did, that there are $N$ doors (where $N > 3$) and that after an initial choice by the contestant MHP will open ($N - 2$) of the remaining doors, leaving your choice and only one other door unopened. Under these circumstances most everyone would make the switch, and this follows from what was recommended from the analytic model and reinforced by the Monte Carlo simulation from the previous section. What if Monty still only opened one door after the contestant's initial choice? Would it still be worthwhile to switch when there are ($N > 3$) doors? This question seems more interesting and highlights a certain diminishing return of Monty's information. Under these rules the probability of switching and winning the prize if instead of 3 doors there were 20 doors is 0.052 or around 5%. More generally, if we let $N$ be the total number of doors and $p$ be the number of doors Monty opens, the probability of winning if the contestant switches is:

$$P(winning \ \& \ switching) = \frac{(N-1)}{N \times (N-p-1)}.$$

For the purpose of the question, it also gives a useful way to visualize the incremental diminishment of the information provided by Monty; as $N$ increases and $p$ stays the same, the knowledge of which door Monty opens is less and less useful to the contestant. Figure 7 visualizes this reduction between the probability for winning of switching compared to not switching in the case $p = 1$ for $N$ ranging from 3 to 20.

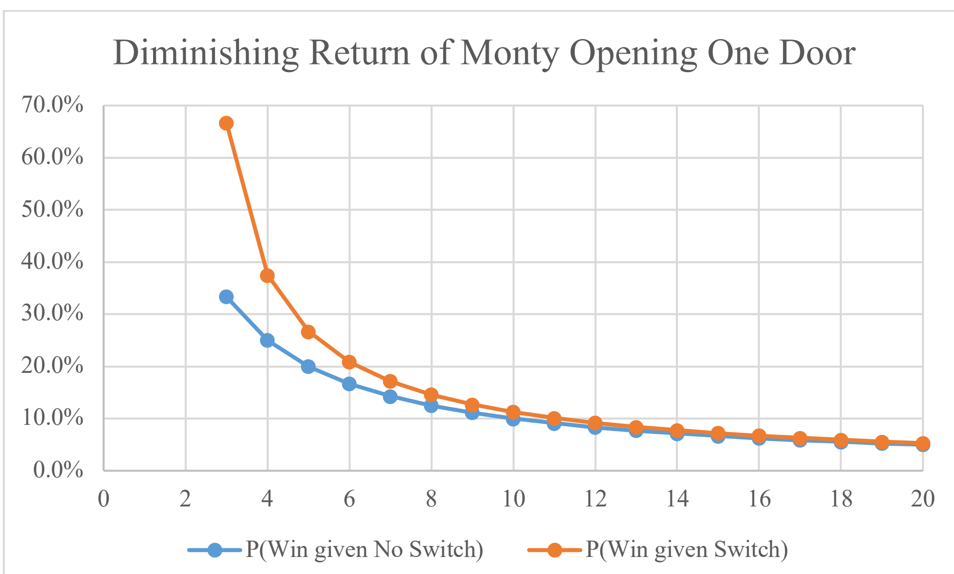


**Figure 7: Diminishing Probability of Monty's Information**

Notice that the above formula confirms the arguments of the traditional MHP. This problem will help students better understand how prior information can be brought into the decision-making process along with the risks of overusing information. The marginal information gained by opening one door will be substantially helpful when there are few doors to pick from (i.e., there is little uncertainty in the contestant's environment). However, when there is considerable uncertainty such as many doors to choose from, the value of the marginal information diminishes greatly. An interesting question to pose to students is to ask how many doors there ought to be such that you would be indifferent between staying with your original choice or switching. Based upon Figure 7, the probability of winning starts to become indistinguishable at around 10 doors or more.

**Conclusions**

Decision making is a challenge that relies on properly taking into account information at each step of the task. We presented a novel use of a commonly used problem to help educate for a better understanding the concepts of conditional probability within decision making. Combining simulation, a powerful tool for deriving solutions when analytic answers are difficult or even impossible, and unintuitive questions educators will be able to aid the development of reliable decision makers.

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# APPENDIX

## Item 1: Cootie Simulation Trial Results

| Standard Game | Initialized Standard Game | Lean Game | Initialized Lean Game |
|---|---|---|---|
| 24 | 14 | 18 | 12 |
| 25 | 14 | 18 | 12 |
| 25 | 15 | 18 | 13 |
| 25 | 15 | 19 | 13 |
| 25 | 15 | 19 | 13 |
| 25 | 15 | 21 | 13 |
| 26 | 16 | 21 | 13 |
| 27 | 16 | 21 | 13 |
| 27 | 17 | 22 | 14 |
| 28 | 17 | 22 | 14 |
| 28 | 17 | 22 | 14 |
| 29 | 17 | 22 | 14 |
| 29 | 18 | 23 | 14 |
| 30 | 18 | 23 | 15 |
| 31 | 18 | 24 | 15 |
| 32 | 18 | 24 | 15 |
| 33 | 18 | 24 | 15 |
| 35 | 18 | 24 | 15 |
| 35 | 18 | 24 | 15 |
| 35 | 18 | 25 | 15 |
| 35 | 18 | 25 | 15 |
| 35 | 19 | 26 | 15 |
| 36 | 19 | 26 | 15 |
| 36 | 19 | 26 | 16 |
| 36 | 19 | 26 | 16 |
| 36 | 19 | 26 | 16 |
| 37 | 19 | 27 | 16 |
| 37 | 20 | 27 | 16 |
| 37 | 20 | 28 | 16 |
| 37 | 20 | 28 | 16 |
| 37 | 21 | 28 | 16 |
| 38 | 21 | 28 | 16 |

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|---|---|---|---|
| 39 | 21 | 28 | 16 |
| 39 | 21 | 29 | 16 |
| 40 | 21 | 29 | 16 |
| 40 | 22 | 29 | 16 |
| 40 | 22 | 30 | 17 |
| 41 | 22 | 30 | 17 |
| 42 | 22 | 30 | 17 |
| 42 | 23 | 30 | 18 |
| 43 | 23 | 31 | 18 |
| 43 | 23 | 31 | 18 |
| 44 | 23 | 32 | 18 |
| 44 | 23 | 33 | 18 |
| 45 | 24 | 33 | 18 |
| 45 | 24 | 33 | 18 |
| 45 | 24 | 34 | 18 |
| 46 | 24 | 34 | 18 |
| 47 | 24 | 35 | 19 |
| 48 | 24 | 35 | 19 |
| 48 | 25 | 35 | 19 |
| 48 | 25 | 35 | 19 |
| 48 | 25 | 37 | 20 |
| 48 | 25 | 37 | 20 |
| 49 | 25 | 37 | 20 |
| 49 | 25 | 37 | 20 |
| 49 | 26 | 37 | 20 |
| 50 | 26 | 37 | 21 |
| 50 | 26 | 37 | 21 |
| 51 | 27 | 38 | 21 |
| 51 | 27 | 38 | 21 |
| 52 | 27 | 40 | 22 |
| 53 | 27 | 41 | 22 |
| 54 | 27 | 41 | 22 |
| 54 | 28 | 42 | 22 |
| 54 | 28 | 43 | 22 |
| 54 | 28 | 44 | 22 |
| 55 | 28 | 44 | 23 |
| 56 | 29 | 45 | 23 |
| 57 | 29 | 46 | 23 |
| 57 | 29 | 46 | 23 |

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|---|---|---|---|
| 58 | 29 | 46 | 24 |
| 59 | 29 | 47 | 24 |
| 59 | 30 | 47 | 24 |
| 59 | 30 | 51 | 24 |
| 59 | 30 | 51 | 26 |
| 60 | 30 | 51 | 27 |
| 60 | 31 | 52 | 27 |
| 60 | 32 | 52 | 29 |
| 61 | 32 | 53 | 29 |
| 61 | 33 | 53 | 29 |
| 62 | 34 | 53 | 29 |
| 62 | 34 | 54 | 30 |
| 63 | 34 | 54 | 30 |
| 64 | 35 | 54 | 32 |
| 65 | 35 | 56 | 33 |
| 66 | 36 | 56 | 33 |
| 66 | 38 | 57 | 34 |
| 66 | 39 | 57 | 34 |
| 68 | 39 | 57 | 36 |
| 68 | 39 | 57 | 37 |
| 68 | 40 | 57 | 38 |
| 71 | 40 | 62 | 40 |
| 71 | 43 | 62 | 42 |
| 72 | 43 | 63 | 45 |
| 74 | 45 | 65 | 45 |
| 82 | 48 | 74 | 49 |
| 84 | 50 | 79 | 50 |
| 88 | 52 | 81 | 55 |
| 104 | 60 | 99 | 69 |

**Item 2: Monty Hall Simulation Table with Seed 1**71

| Contest Picks | Car Behind Door | Monty Opens Door | Player Switches to Door | Player Wins | Cumulative Wins Not Switching | Cumulative Wins Switching |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | Car | 1 | 0 |
| 1 | 2 | 3 | 2 | Cow | 0.5 | 0.5 |
| 1 | 1 | 2 | 3 | Cow | 0.666666667 | 0.33333333 |
| 1 | 2 | 3 | 2 | Cow | 0.5 | 0.5 |
| 1 | 1 | 2 | 3 | Cow | 0.6 | 0.4 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 2 | Cow | 0.5 | 0.5 |
| 1 | 1 | 2 | 3 | Cow | 0.571428571 | 0.42857143 |
| 1 | 1 | 2 | 3 | Cow | 0.625 | 0.375 |
| 1 | 1 | 2 | 3 | Cow | 0.666666667 | 0.33333333 |
| 1 | 3 | 2 | 3 | Cow | 0.6 | 0.4 |
| 1 | 2 | 3 | 2 | Cow | 0.545454545 | 0.45454545 |
| 1 | 3 | 2 | 3 | Cow | 0.5 | 0.5 |
| 1 | 1 | 2 | 3 | Cow | 0.538461538 | 0.46153846 |
| 1 | 3 | 2 | 3 | Cow | 0.5 | 0.5 |
| 1 | 1 | 2 | 3 | Cow | 0.533333333 | 0.46666667 |
| 1 | 2 | 3 | 2 | Cow | 0.5 | 0.5 |
| 1 | 3 | 2 | 3 | Cow | 0.470588235 | 0.52941176 |
| 1 | 1 | 2 | 3 | Cow | 0.5 | 0.5 |
| 1 | 2 | 3 | 2 | Cow | 0.473684211 | 0.52631579 |
| 1 | 2 | 3 | 2 | Cow | 0.45 | 0.55 |
| 1 | 2 | 3 | 2 | Cow | 0.428571429 | 0.57142857 |
| 1 | 2 | 3 | 2 | Cow | 0.409090909 | 0.59090909 |
| 1 | 3 | 2 | 3 | Cow | 0.391304348 | 0.60869565 |
| 1 | 1 | 2 | 3 | Cow | 0.416666667 | 0.58333333 |
| 1 | 3 | 2 | 3 | Cow | 0.4 | 0.6 |
| 1 | 1 | 2 | 3 | Cow | 0.423076923 | 0.57692308 |
| 1 | 3 | 2 | 3 | Cow | 0.407407407 | 0.59259259 |
| 1 | 3 | 2 | 3 | Cow | 0.392857143 | 0.60714286 |
| 1 | 1 | 2 | 3 | Cow | 0.413793103 | 0.5862069 |
| 1 | 3 | 2 | 3 | Cow | 0.4 | 0.6 |
| 1 | 3 | 2 | 3 | Cow | 0.387096774 | 0.61290323 |
| 1 | 2 | 3 | 2 | Cow | 0.375 | 0.625 |
| 1 | 3 | 2 | 3 | Cow | 0.363636364 | 0.63636364 |
| 1 | 1 | 2 | 3 | Cow | 0.382352941 | 0.61764706 |
| 1 | 3 | 2 | 3 | Cow | 0.371428571 | 0.62857143 |
| 1 | 1 | 2 | 3 | Cow | 0.361111111 | 0.61111111 |
| 1 | 1 | 2 | 3 | Cow | 0.378378378 | 0.59459459 |
| 1 | 2 | 3 | 2 | Cow | 0.368421053 | 0.60526316 |
| 1 | 2 | 3 | 2 | Cow | 0.358974359 | 0.61538462 |
| 1 | 2 | 3 | 2 | Cow | 0.35 | 0.625 |
| 1 | 1 | 2 | 3 | Cow | 0.365853659 | 0.6097561 |
| 1 | 1 | 2 | 3 | Cow | 0.380952381 | 0.5952381 |
| 1 | 2 | 3 | 2 | Cow | 0.372093023 | 0.60465116 |
| 1 | 3 | 2 | 3 | Cow | 0.363636364 | 0.61363636 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 2 | Cow | 0.355555556 | 0.62222222 |
| 1 | 2 | 3 | 2 | Cow | 0.347826087 | 0.63043478 |
| 1 | 1 | 2 | 3 | Cow | 0.361702128 | 0.61702128 |
| 1 | 1 | 2 | 3 | Cow | 0.375 | 0.60416667 |
| 1 | 2 | 3 | 2 | Cow | 0.367346939 | 0.6122449 |
| 1 | 2 | 3 | 2 | Cow | 0.36 | 0.62 |
| 1 | 1 | 2 | 3 | Cow | 0.37254902 | 0.60784314 |
| 1 | 3 | 2 | 3 | Cow | 0.365384615 | 0.61538462 |
| 1 | 2 | 3 | 2 | Cow | 0.358490566 | 0.62264151 |
| 1 | 1 | 2 | 3 | Cow | 0.37037037 | 0.61111111 |
| 1 | 2 | 3 | 2 | Cow | 0.363636364 | 0.61818182 |
| 1 | 2 | 3 | 2 | Cow | 0.357142857 | 0.625 |
| 1 | 1 | 2 | 3 | Cow | 0.368421053 | 0.61403509 |
| 1 | 2 | 3 | 2 | Cow | 0.362068966 | 0.62068966 |
| 1 | 2 | 3 | 2 | Cow | 0.355932203 | 0.62711864 |
| 1 | 2 | 3 | 2 | Cow | 0.35 | 0.63333333 |
| 1 | 3 | 2 | 3 | Cow | 0.344262295 | 0.63934426 |
| 1 | 1 | 2 | 3 | Cow | 0.35483871 | 0.62903226 |
| 1 | 1 | 2 | 3 | Cow | 0.365079365 | 0.61904762 |
| 1 | 2 | 3 | 2 | Cow | 0.359375 | 0.625 |
| 1 | 3 | 2 | 3 | Cow | 0.353846154 | 0.63076923 |
| 1 | 2 | 3 | 2 | Cow | 0.348484848 | 0.63636364 |
| 1 | 2 | 3 | 2 | Cow | 0.343283582 | 0.64179104 |
| 1 | 2 | 3 | 2 | Cow | 0.338235294 | 0.64705882 |
| 1 | 1 | 2 | 3 | Cow | 0.347826087 | 0.63768116 |
| 1 | 2 | 3 | 2 | Cow | 0.342857143 | 0.64285714 |
| 1 | 2 | 3 | 2 | Cow | 0.338028169 | 0.64788732 |
| 1 | 3 | 2 | 3 | Cow | 0.333333333 | 0.65277778 |
| 1 | 2 | 3 | 2 | Cow | 0.328767123 | 0.65753425 |
| 1 | 1 | 2 | 3 | Cow | 0.337837838 | 0.64864865 |
| 1 | 2 | 3 | 2 | Cow | 0.333333333 | 0.65333333 |
| 1 | 2 | 3 | 2 | Cow | 0.328947368 | 0.65789474 |
| 1 | 3 | 2 | 3 | Cow | 0.324675325 | 0.66233766 |
| 1 | 2 | 3 | 2 | Cow | 0.320512821 | 0.66666667 |
| 1 | 1 | 2 | 3 | Cow | 0.329113924 | 0.65822785 |
| 1 | 3 | 2 | 3 | Cow | 0.325 | 0.6625 |
| 1 | 2 | 3 | 2 | Cow | 0.320987654 | 0.66666667 |
| 1 | 2 | 3 | 2 | Cow | 0.317073171 | 0.67073171 |
| 1 | 3 | 2 | 3 | Cow | 0.313253012 | 0.6746988 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 3 | Cow | 0.30952381 | 0.67857143 |
| 1 | 2 | 3 | 2 | Cow | 0.305882353 | 0.68235294 |
| 1 | 2 | 3 | 2 | Cow | 0.302325581 | 0.68604651 |
| 1 | 3 | 2 | 3 | Cow | 0.298850575 | 0.68965517 |
| 1 | 1 | 2 | 3 | Cow | 0.306818182 | 0.68181818 |
| 1 | 3 | 2 | 3 | Cow | 0.303370787 | 0.68539326 |
| 1 | 3 | 2 | 3 | Cow | 0.3 | 0.68888889 |
| 1 | 3 | 2 | 3 | Cow | 0.296703297 | 0.69230769 |
| 1 | 2 | 3 | 2 | Cow | 0.293478261 | 0.69565217 |
| 1 | 1 | 2 | 3 | Cow | 0.301075269 | 0.68817204 |
| 1 | 3 | 2 | 3 | Cow | 0.29787234 | 0.69148936 |
| 1 | 1 | 2 | 3 | Cow | 0.305263158 | 0.68421053 |
| 1 | 3 | 2 | 3 | Cow | 0.302083333 | 0.6875 |
| 1 | 2 | 3 | 2 | Cow | 0.298969072 | 0.69072165 |
| 1 | 3 | 2 | 3 | Cow | 0.295918367 | 0.69387755 |
| 1 | 1 | 2 | 3 | Cow | 0.303030303 | 0.68686869 |
| 1 | 1 | 2 | 3 | Cow | 0.31 | 0.68 |

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**Item 2: Monty Hall Simulation Table with Seed 17**

| Contest Picks | Car Behind Door | Monty Opens | Player Switches to Door | Player Wins | Cumulative Wins Not Switching | Cumulative Wins Switching |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | Cow | 1 | 0 |
| 1 | 1 | 2 | 3 | Cow | 1 | 0 |
| 1 | 3 | 2 | 3 | Car | 0.666667 | 0.333333 |
| 1 | 3 | 2 | 3 | Car | 0.5 | 0.5 |
| 1 | 1 | 2 | 3 | Cow | 0.6 | 0.4 |
| 1 | 3 | 2 | 3 | Car | 0.5 | 0.5 |
| 1 | 3 | 2 | 3 | Car | 0.428571 | 0.571429 |
| 1 | 2 | 3 | 2 | Car | 0.375 | 0.625 |
| 1 | 2 | 3 | 2 | Car | 0.333333 | 0.666667 |
| 1 | 3 | 2 | 3 | Car | 0.3 | 0.7 |
| 1 | 2 | 3 | 2 | Car | 0.272727 | 0.727273 |
| 1 | 3 | 2 | 3 | Car | 0.25 | 0.75 |
| 1 | 1 | 2 | 3 | Cow | 0.307692 | 0.692308 |
| 1 | 2 | 3 | 2 | Car | 0.285714 | 0.714286 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 3 | Car | 0.266667 | 0.733333 |
| 1 | 1 | 2 | 3 | Cow | 0.3125 | 0.6875 |
| 1 | 2 | 3 | 2 | Car | 0.294118 | 0.705882 |
| 1 | 1 | 2 | 3 | Cow | 0.333333 | 0.666667 |
| 1 | 2 | 3 | 2 | Car | 0.315789 | 0.684211 |
| 1 | 1 | 2 | 3 | Cow | 0.35 | 0.65 |
| 1 | 1 | 2 | 3 | Cow | 0.380952 | 0.619048 |
| 1 | 3 | 2 | 3 | Car | 0.363636 | 0.636364 |
| 1 | 1 | 2 | 3 | Cow | 0.391304 | 0.608696 |
| 1 | 3 | 2 | 3 | Car | 0.375 | 0.625 |
| 1 | 2 | 3 | 2 | Car | 0.36 | 0.64 |
| 1 | 3 | 2 | 3 | Car | 0.346154 | 0.653846 |
| 1 | 2 | 3 | 2 | Car | 0.333333 | 0.666667 |
| 1 | 1 | 2 | 3 | Cow | 0.357143 | 0.642857 |
| 1 | 1 | 2 | 3 | Cow | 0.37931 | 0.62069 |
| 1 | 3 | 2 | 3 | Car | 0.366667 | 0.633333 |
| 1 | 2 | 3 | 2 | Car | 0.354839 | 0.645161 |
| 1 | 3 | 2 | 3 | Car | 0.34375 | 0.65625 |
| 1 | 2 | 3 | 2 | Car | 0.333333 | 0.666667 |
| 1 | 3 | 2 | 3 | Car | 0.323529 | 0.676471 |
| 1 | 2 | 3 | 2 | Car | 0.314286 | 0.685714 |
| 1 | 2 | 3 | 2 | Car | 0.305556 | 0.694444 |
| 1 | 3 | 2 | 3 | Car | 0.297297 | 0.702703 |
| 1 | 3 | 2 | 3 | Car | 0.289474 | 0.710526 |
| 1 | 3 | 2 | 3 | Car | 0.282051 | 0.717949 |
| 1 | 1 | 2 | 3 | Cow | 0.3 | 0.7 |
| 1 | 1 | 2 | 3 | Cow | 0.317073 | 0.682927 |
| 1 | 3 | 2 | 3 | Car | 0.309524 | 0.690476 |
| 1 | 1 | 2 | 3 | Cow | 0.325581 | 0.674419 |
| 1 | 3 | 2 | 3 | Car | 0.318182 | 0.681818 |
| 1 | 1 | 2 | 3 | Cow | 0.333333 | 0.666667 |
| 1 | 1 | 2 | 3 | Cow | 0.347826 | 0.652174 |
| 1 | 3 | 2 | 3 | Car | 0.340426 | 0.659574 |
| 1 | 1 | 2 | 3 | Cow | 0.354167 | 0.645833 |
| 1 | 2 | 3 | 2 | Car | 0.346939 | 0.653061 |
| 1 | 2 | 3 | 2 | Car | 0.34 | 0.66 |
| 1 | 1 | 2 | 3 | Cow | 0.352941 | 0.647059 |
| 1 | 1 | 2 | 3 | Cow | 0.365385 | 0.634615 |
| 1 | 1 | 2 | 3 | Cow | 0.377358 | 0.622642 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 3 | Car | 0.37037 | 0.62963 |
| 1 | 1 | 2 | 3 | Cow | 0.381818 | 0.618182 |
| 1 | 3 | 2 | 3 | Car | 0.375 | 0.625 |
| 1 | 1 | 2 | 3 | Cow | 0.385965 | 0.614035 |
| 1 | 1 | 2 | 3 | Cow | 0.396552 | 0.603448 |
| 1 | 2 | 3 | 2 | Car | 0.389831 | 0.610169 |
| 1 | 1 | 2 | 3 | Cow | 0.4 | 0.6 |
| 1 | 2 | 3 | 2 | Car | 0.393443 | 0.606557 |
| 1 | 2 | 3 | 2 | Car | 0.387097 | 0.612903 |
| 1 | 3 | 2 | 3 | Car | 0.380952 | 0.619048 |
| 1 | 2 | 3 | 2 | Car | 0.375 | 0.625 |
| 1 | 3 | 2 | 3 | Car | 0.369231 | 0.630769 |
| 1 | 2 | 3 | 2 | Car | 0.363636 | 0.636364 |
| 1 | 2 | 3 | 2 | Car | 0.358209 | 0.641791 |
| 1 | 2 | 3 | 2 | Car | 0.352941 | 0.647059 |
| 1 | 1 | 2 | 3 | Cow | 0.362319 | 0.637681 |
| 1 | 1 | 2 | 3 | Cow | 0.371429 | 0.628571 |
| 1 | 2 | 3 | 2 | Car | 0.366197 | 0.633803 |
| 1 | 2 | 3 | 2 | Car | 0.361111 | 0.638889 |
| 1 | 3 | 2 | 3 | Car | 0.356164 | 0.643836 |
| 1 | 2 | 3 | 2 | Car | 0.351351 | 0.648649 |
| 1 | 1 | 2 | 3 | Cow | 0.36 | 0.64 |
| 1 | 3 | 2 | 3 | Car | 0.355263 | 0.644737 |
| 1 | 3 | 2 | 3 | Car | 0.350649 | 0.649351 |
| 1 | 3 | 2 | 3 | Car | 0.346154 | 0.653846 |
| 1 | 3 | 2 | 3 | Car | 0.341772 | 0.658228 |
| 1 | 1 | 2 | 3 | Cow | 0.35 | 0.65 |
| 1 | 1 | 2 | 3 | Cow | 0.358025 | 0.641975 |
| 1 | 1 | 2 | 3 | Cow | 0.365854 | 0.634146 |
| 1 | 1 | 2 | 3 | Cow | 0.373494 | 0.626506 |
| 1 | 3 | 2 | 3 | Car | 0.369048 | 0.630952 |
| 1 | 3 | 2 | 3 | Car | 0.364706 | 0.635294 |
| 1 | 3 | 2 | 3 | Car | 0.360465 | 0.639535 |
| 1 | 2 | 3 | 2 | Car | 0.356322 | 0.643678 |
| 1 | 2 | 3 | 2 | Car | 0.352273 | 0.647727 |
| 1 | 3 | 2 | 3 | Car | 0.348315 | 0.651685 |
| 1 | 1 | 2 | 3 | Cow | 0.355556 | 0.644444 |
| 1 | 3 | 2 | 3 | Car | 0.351648 | 0.648352 |
| 1 | 1 | 2 | 3 | Cow | 0.358696 | 0.641304 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 3 | Car | 0.354839 | 0.645161 |
| 1 | 3 | 2 | 3 | Car | 0.351064 | 0.648936 |
| 1 | 1 | 2 | 3 | Cow | 0.357895 | 0.642105 |
| 1 | 2 | 3 | 2 | Car | 0.354167 | 0.645833 |
| 1 | 1 | 2 | 3 | Cow | 0.360825 | 0.639175 |
| 1 | 1 | 2 | 3 | Cow | 0.367347 | 0.632653 |
| 1 | 2 | 3 | 2 | Car | 0.363636 | 0.636364 |
| 1 | 1 | 2 | 3 | Cow | 0.37 | 0.63 |